

# ALGEBRA OF LIGHT: MULTIPLICATION AND DIVISION OF ORBITAL ANGULAR MOMENTUM

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*A novel and compact solution is presented to efficiently multiply or divide the orbital angular momentum (OAM) of light beams using a sequence of two phase-elements implementing a properly designed superposition of circular-sector transformations. The experimental tests confirm the capability of the multiplier optics to perform integer multiplication of the input OAM, while the designed dividers are demonstrated to correctly split up the input beam into a complementary set of OAM beams. These elements can find promising applications for the multiplicative generation of higher-order OAM modes, optical information processing based on OAM-beams transmission, and optical routing/switching in telecom.*

**Keywords:** orbital angular momentum, transformation optics, multiplication, division, electron-beam lithography

In the last decades, light beams carrying OAM gave rise to a flourishing field of research, leading to a rich multiplicity of studies and applications [1]. In the telecom field, in particular, the potentially unbounded state space provided by this even-unexploited degree of freedom opens to a promising solution to increase the information capacity of optical networks [2], both for free-space and optical fibre propagation. Nowadays, it becomes urgent to further develop novel devices that can reconfigure and switch between distinct OAM modes dynamically, in order to fully exploit the extra degree of freedom provided by the OAM for both classical and quantum communications.

So far, OAM multiplication and division have been implemented by means of bulky and sophisticated solutions [3, 4], based on the *log-pol* optical transformation, which are barely suitable to integration and miniaturization, due to the presence of many optical elements. We have recently developed a completely different method [5], which basically preserves the axial symmetry and avoids the limitations of the coordinate-change approach. The key-element is represented by an optical transformation which maps the azimuthal phase gradient of the input OAM beams into a circular sector. By combining multiple circular-sector transformations onto a single optical element (Fig. 1(a)), it is possible to perform the multiplication of the input OAM by mapping its phase onto complementary circular sectors. Conversely, by combining multiple inverse transformations, it is possible to map different complementary sectors of the input beam into an equal number of circular phase gradients, thus achieving a division of the initial OAM. These operations can be realized by a sequence of only two elements, performing the optical transformation of the beam and the required phase-correction, respectively. This approach allows to perform multiplication and division of OAM in a compact manner, remarkably reducing the number of optical operations and the total amount of optical elements, and therefore providing a final significant increase in the optical efficiency.

The designed optical elements have been fabricated in the form of miniaturized phase-only diffractive optics (Fig. 1(b-d)) with high-resolution electron-beam lithography [6], and optically characterized in order to demonstrate the expected capability

to either multiply or divide the OAM of the input beam, in case of two-fold and three-fold OAM division/multiplication. A compact optical architecture has been presented in order to arrange the two optical elements, required for optical transformation and phase correction, onto the same substrate, further improving alignment and miniaturization. Moreover, a generalized version is presented and tested, performing combined division and multiplication over an arbitrary set of values in a compact and efficient way.

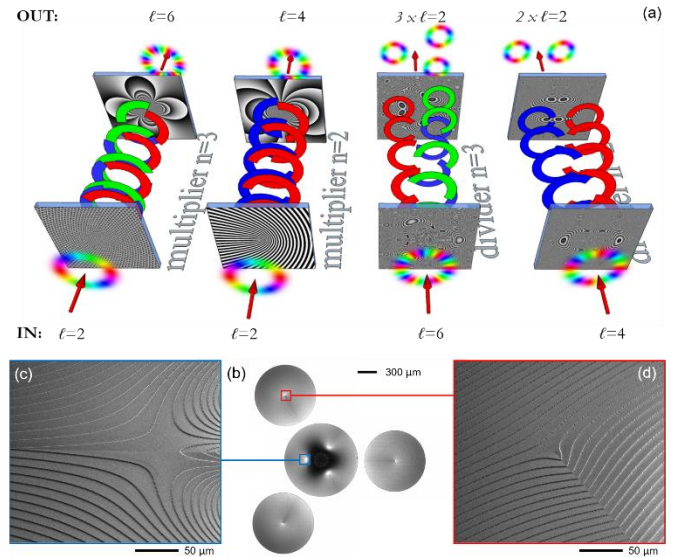


Fig. 1 (a) Scheme of 3-fold and 2-fold OAM multiplication and division. (b) Diffractive optics for compact 3-fold division and details at SEM (c, d).

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